Decentralized Dynamic Allocation of Subchannels in Multiple Access Networks

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***Abstract*—This letter introduces and analyzes a new distributed dynamic subchannel allocation algorithm for multiple access networks. The proposed algorithm is based on general order selection, and exhibits low complexity and low processing delay. The exact and asymptotic closed-form expressions for the average error probability are derived, and it is shown that there is a significant system performance advantage over the conventional uniform allocation approach. Numerical studies also show that the performance of our new algorithm can be close to that of the highly complex optimal search method.**

***Index Terms*—Channel allocation, diversity, fading channels, general order selection, multiple access.**

I. Introduction

A practical approach to spectrum allocation in distributed wireless networks is for each user to transmit in a narrow band, and avoid other users by a combination of traditional carrier-sense and frequency-division multiple access schemes. One popular approach is the uniform allocation (UA) scheme [1], whereby each user randomly (i.e. uniformly) selects a single narrow band "subchannel" for its transmission, from the available (i.e. unoccupied) set of subchannels. The scheme is distributed in nature, and has low overhead. However, it can suffer from a high outage rate, since it does not take channel quality into consideration in the choice of the subchannel. As such, there is nothing stopping deeply faded subchannels from being selected, when stronger ones are available.

In this letter we propose a modification to the UA scheme which takes channel quality into consideration. We view the subchannel selection problem from the viewpoint of diversity combining, and in particular as an explicit case of selection combining (SC) [2]. Traditional SC involves using only the subchannel with the highest gain. In the multi-access wireless network case, we propose to generalize SC by selecting the subchannel with the highest gain, but restricting the selection to the subset of subchannels not already being used by other users. This can be viewed as an example of general order selection (GOS) [3, 4], and hence we call our new algorithm the GOS-based allocation (GOSA) algorithm.

The performance of our new GOSA algorithm is analyzed both analytically and numerically. We derive an exact analytical bit error rate (BER) expression for a network with an arbitrary number of users. An asymptotic expression is also attained to explicitly reveal the relationship between the diversity order and the number of users and subchannels. The exact and asymptotic BER expressions are also developed for the standard UA algorithm. It is shown that GOSA can offer significant performance improvements over UA. Moreover, we numerically compare the performance of GOSA with an optimal exhaustive search allocation (ESA) method [5] (which is of course prohibitively complex for practical implementation), and show that the performance of GOSA is close to that of ESA.

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II. GOS for Subchannel Allocation: Proposed GOSA Algorithm

Consider a typical multiple access communication system such as an ad hoc network in which K users transmit over L subchannels ($K \leq L$). In our proposed GOSA algorithm, each user transmits on a single subchannel and acquires its own subchannel in a distributed fashion by selecting its *highest available* gain subchannel. The term *highest available* implies a free subchannel for which its gain is highest for that user. The number of users accessing and/or in the network at any particular point in time is variable, and the users randomly access the network without a predetermined policy.

The GOSA algorithm is outlined as follows. The transmitter identifies available subchannels via carrier-sensing. The receiver obtains CSI of the available subchannels, estimated based on a pilot sequence sent by the transmitter in each available subchannel. The receiver identifies and selects the subchannel with the highest gain, from the available subchannels. The receiver then feeds back the index of the selected subchannel to the transmitter, and transmission commences. Note that for both UA and GOSA, users require knowledge of subchannel availability and CSI to enable coherent demodulation. The UA receiver only requires CSI of the randomly selected subchannel, while the GOSA receiver requires CSI of all the available subchannels. However, for both systems, the receiver only feeds back the index of the selected subchannel to the transmitter. Furthermore, because of the distributed non-coordinated (no communication among users) and non-iterative nature of our new GOSA algorithm, the amount of overhead and processing delays involved in selecting the favorable subchannel is low. In Sections III and IV, we demonstrate by both analysis and simulation that GOSA achieves significant performance improvement at the cost of this relatively small overhead. This makes GOSA highly desirable for delay-constrained applications that can demand high data rates in fast-varying wireless communication channels.

An illustrative example of the GOSA approach is shown in Fig. 1, for a network of $K = 7$ users and $L = 8$ subchannels. In the figure, $G_{\ell}^{(k)}$ denotes the gain of subchannel ℓ of user k and $G_{r:L}^{(k)}$ denotes the subchannel with the r-th highest gain out of L subchannels of user k. Previous users

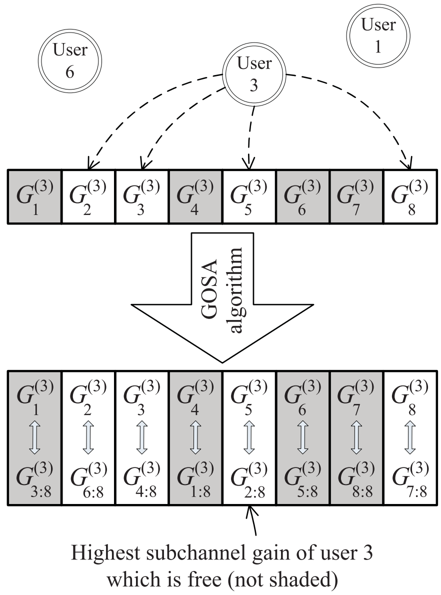

Fig. 1. An example of the proposed GOSA algorithm.

that have accessed the network and acquired subchannels are indicated by the shaded zones (such subchannels are marked as unavailable for new users accessing the network). In this example, users 1, 3, and 6 are awaiting network access. User 3 contends for network access and identifies subchannel 5 as its *highest available* gain subchannel, which is then selected for transmission. Subchannel 5 will not be available for subsequent users accessing the network (i.e., users 1 and 6).

III. Exact and Asymptotic BER Evaluation of GOSA Algorithm

We now analyze the performance of GOSA in terms of the BER. It is assumed that the signal on each subchannel experiences independent identically distributed (iid) Rayleigh fading. Let $G_1^{(k)}, G_2^{(k)}, \cdots, G_L^{(k)}$ be L iid random variables corresponding to the subchannel gains of arbitrary user k. The corresponding subchannel signal-to-noise ratios (SNR's) are denoted by

$$\Gamma_\ell^{(k)} \triangleq [G_\ell^{(k)}]^2 E_b/N_0, \quad \ell = 1, 2, \ldots, L \tag{1}$$

where E_b is the transmitted bit energy and N_0 is the one-sided noise power spectral density. Without loss of generality, it suffices to consider one of the users and hence omit the superscript. The instantaneous SNR, Γ, on each subchannel is distributed according to an exponential distribution. Hence, the common probability density function (pdf) and cumulative distribution function (cdf) are given by $f_\Gamma(\gamma) = \frac{1}{\gamma_0} e^{-\frac{\gamma}{\gamma_0}}$ and $F_\Gamma(\gamma) = 1 - e^{-\gamma/\gamma_0}$, respectively, where γ_0 is the common average value of Γ. $G_1, \cdots, G_L$ are arranged in decreasing order of their magnitudes such that $G_{1:L} \geq \cdots \geq G_{L:L}$. Accordingly, the r-th highest SNR is

$$\Gamma_{r:L} = [G_{r:L}]^2 E_b/N_0. \tag{2}$$

The pdf for the r-th highest SNR can be obtained using a standard result in order statistics [6] assuming that a random sample of size L is drawn from $f_\Gamma(\gamma)$ and can be written as

$$\begin{aligned} f_{\Gamma_{r:L}}(\gamma) &= \frac{L!}{(L-r)!(r-1)!} \left[F_\Gamma(\gamma)\right]^{L-r} \left[1 - F_\Gamma(\gamma)\right]^{r-1} f_\Gamma(\gamma) \\ &= \frac{L!}{(L-r)!(r-1)!} \left(1 - e^{-\gamma/\gamma_0}\right)^{L-r} \times \frac{1}{\gamma_0} e^{-r\gamma/\gamma_0}. \end{aligned} \tag{3}$$

Consequently, the average bit error probability when transmitting on the subchannel with the r-th highest gain (out of L subchannels), denoted by $P_{r:L}$, can be calculated as

$$P_{r:L} = \int_0^\infty P(e|\gamma) f_{\Gamma_{r:L}}(\gamma) d\gamma. \tag{4}$$

For binary phase shift keying (BPSK), after substituting $P(e|\gamma) = Q(\sqrt{2\gamma})$ and (3) into (4), we get

$$\begin{aligned} P_{r:L} = \frac{L!}{2(L-r)!(r-1)!} &\sum_{j=0}^{L-r} \frac{(-1)^j}{r+j} \binom{L-r}{j} \\ &\times \left[1 - \sqrt{\frac{\gamma_0}{\gamma_0 + r + j}}\right]. \end{aligned} \tag{5}$$

The BER for the GOSA algorithm, P_{GOSA}, can be expressed in terms of $P_{r:L}$. We first calculate the probability of error for the user that selects its subchannel after $k-1$ users have already made their selections, and then the BER is found by averaging over k as follows

$$P_{\text{GOSA}} = \frac{1}{K} \sum_{k=1}^{K} \sum_{r=1}^{k} P_{r:L} X_{r,k}, \tag{6}$$

where $X_{r,k}$ is the probability that the r-th highest gain subchannel is available to the k-th user accessing the network, given that $k-1$ subchannels have been allocated to other users and are marked as unavailable. It can be shown that $X_{r,k}$ has the following general form

$$X_{r,k} = \begin{cases} \frac{L-k+1}{L}\,, & r = 1 \\ (r-1)! \begin{pmatrix} k-1 \\ r-1 \end{pmatrix} \prod\limits_{i=0}^{k-2} \frac{1}{L-i} \prod\limits_{j=r}^{k-1} (L-j)\,, & 1 < r < k \\ 1 - \sum\limits_{r=1}^{k-1} X_{r,k}\,, & r = k\,. \end{cases} \tag{7}$$

At high SNR, it is possible to take the Taylor series expansion of (6) and use the dominating term to give an expression for P_{GOSA} as $\gamma_0 \to \infty$, as follows

$$\begin{aligned} P_{\text{GOSA}}^\infty = \frac{1}{K} \cdot \frac{[2(L-K+1)-1]!}{2^{2(L-K+1)}(L-K)!} &{\gamma_0}^{-(L-K+1)} \\ &+ o\left({\gamma_0}^{-(L-K+1)}\right), \end{aligned} \tag{8}$$

where we write $f(x) = o\,[g(x)]$, $x \to x_0$, if $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$. It is clearly shown that an asymptotic diversity order of $L-K+1$ is achieved at high SNR, which clearly reveals the impact of the number of subchannels and the number of users on the system error performance.

The error performance of UA is included for comparison. In this case, subchannel selection is across all the available subchannels and the selection across the L subchannels is of equal chance. The BER expression for UA is independent of

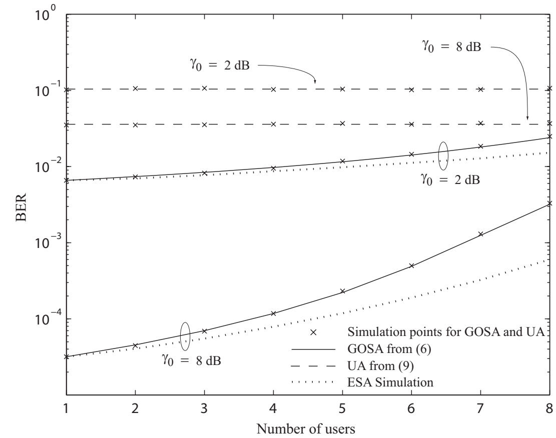

Fig. 2. BER of GOSA, UA, and ESA for γ_0 = 2 and 8 dB and $L = 8$ subchannels, for BPSK.

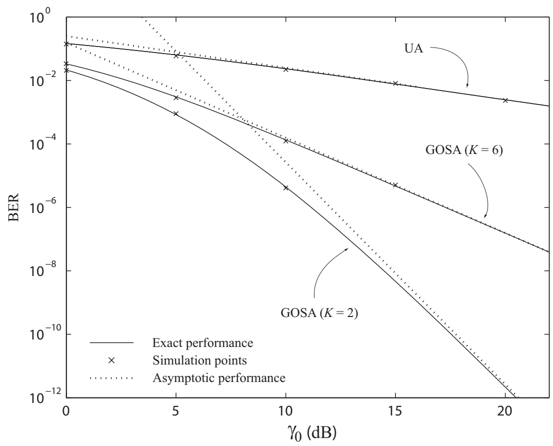

Fig. 3. Performance comparison of exact and asymptotic results for K = 2 and 6 users and $L = 8$ subchannels, for BPSK.

the number of users, K, and may readily be obtained as a function of $P_{r:L}$ as

$$P_{\text{UA}} = \frac{1}{L}\sum_{r=1}^{L} P_{r:L}. \tag{9}$$

In this case, we can show that, at high SNR, (9) reduces to

$$P_{\text{UA}}^{\infty} = \frac{1}{4}\gamma_0^{-1} + o\left(\gamma_0^{-1}\right). \tag{10}$$

A comparison between (8) and (10) clearly indicates the diversity advantage of GOSA over UA when $K < L$. For a fully loaded system where $L = K$, (8) reduces to

$$P_{\text{GOSA}}^{\infty} = \frac{1}{4K}\gamma_0^{-1} + o\left(\gamma_0^{-1}\right). \tag{11}$$

By comparing (10) and (11), we find that GOSA has SNR advantage over UA despite the same diversity order.

IV. Numerical Results

The BER of GOSA is compared with UA and ESA in Fig. 2 for BPSK modulation. The curves are plotted using (6) and (9) for an average SNR of $\gamma_0 = 2$ and 8 dB assuming that the $L = 8$ subchannels undergo independent Rayleigh fading. The results show that GOSA can provide a much lower BER than UA over a wide range of the number of users. We also show simulation results to highlight the accuracy of the analysis.

It is evident that as γ_0 increases, so does the improvement offered by GOSA compared to UA. At $\gamma_0 = 2$ dB for a lightly loaded network with $K = 2$ users, it is observed that GOSA can yield a 14-fold reduction in BER compared to UA. At $\gamma_0 = 8$ dB, GOSA yields a 780-fold reduction in BER. For a fully loaded network with $K = 8$ users, the performance difference between GOSA and UA is roughly 4-fold and 12-fold when operating at $\gamma_0 = 2$ and 8 dB, respectively.

To compare the performance of GOSA with an optimal allocation scheme, ESA is considered. ESA examines the set of all possible $L!/(L-K)!$ subchannel allocation permutations across all users and selects the one that offers the lowest BER. While this is unrealistic in a practical scenario, ESA provides important performance benchmark. We see that while GOSA is more complex than UA, it is nowhere near the computational complexity of ESA. Note in Fig. 2, for a lower SNR, how the BER of GOSA approaches that of ESA over a wide range of the number of users. We also find that GOSA performs close to ESA when the network is lightly loaded. This is due to the fact that higher order subchannel gains (i.e., stronger) are considered. It can also be seen that the performance difference between GOSA and ESA slightly increases with the number of users. This is what we expect since larger K implies that there is a higher chance that a lower order gain subchannel (i.e., weaker) is considered.

Fig. 3 shows curves for GOSA and UA as a function of SNR. The respective "exact performance" curves come from (6) and (9), and the respective "asymptotic performance" curves come from (8) and (10). The figure clearly shows that the analytical results match the simulation ones, and that the exact expressions approach the asymptotic ones at high SNR, which are asymptotically tight bounds of exact expressions.

V. Conclusion

We proposed and analyzed a new distributed practical algorithm that dynamically allocates subchannels in a multiuser network. The decision is based on each user's local instantaneous wireless channel conditions. Error probability calculations and simulation results demonstrated that our new GOSA algorithm can provide a much lower BER than UA, and performs close to the optimal ESA method over a wide range of the number of users, but with significantly lighter computational burden.

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